KEYS TO CALCULUS PRIMER PROBLEMS

1.1) $Y=8X$

$dY/dX=8$

1.2) $Y=3X^{-1}$

$dY/dX=-3X^{-2}=3/X^2$

1.3) $Y=100X-6X^2$

$dY/dX=100-12X$

1.4) $Y=-100x + 12x^2 - .75x^3$

$dY/dx=-100 + 24x - 2.25x^2$

1.5) $Y= 5*X^{.5}$

$dY/dX=2.5X^{-.5}= 2.5/\sqrt{X}$

1.6) $Y=1/x^{.33}$

$dY/dX=1/3*(X^{-.33-1})= 1/3*(X^{-1.33})= 1/3X^{1.33}$

1.7) $Y=e^{-x}$

$dY/dX= e^{-x}$

1.8) $Y=\ln X$

$dY/dX=1/X$

1.9) $Y=8x*9x^2$

$dY/dx=8x * 18x + 8*9x^2 = 144\ x^2 + 72\ x^2 = 216\ x^2$

1.10) $Y=4X/3X^2$

$$dY/dX=\frac{-4*3X^2 - 4X*6X}{(3X^2)^2}=-12/9X^2=-4/3X^2$$

1.11) $Y=100x^3-6x^2$

$dY/dX=300X^2-12X$

1.12) $Y=-10Z^2$ and $Z=7X$

$dY/dX = dY/dZ * dZ/dX=-20Z*7=-140Z=-140*7X=-980X$

2.1) $Y=X + X*Z - Z^2$
$\partial Y/\partial X=1+Z$
$\partial Y/\partial Z=X-2Z$

2.2) $Y=40 + 3X + 4Z^2$
$\partial Y/\partial X=3$
$\partial Y/\partial Z=8Z$

2.3) $Y=6X^3 + 4*X*Z + Z^2$
$\partial Y/\partial X=18X^2 + 4Z$
$\partial Y/\partial Z=4X + 2Z$

2.4) $Y=(4X + 10Z)*(.5X + 2Z)$ **(Use the product rule here!!)**

$\partial Y/\partial X=(4X+10Z)*.5 + 4*(.5X+2Z)=4X+13Z$

$\partial Y/\partial Z=(4X+10Z)*2 + 10*(.5X+2Z)=13X + 40Z$

2.5) $Y=(4X + 10Z)/(.5X + 2Z)$ **(Use the quotient rule here!!)**

$$\partial Y/\partial X= \frac{4*(.5X+2Z)-(4X+10Z)*.5}{(5X+2Z)^2} = -3Z/(5X+2Z)^2$$

$$\partial Y/\partial Z=\frac{10*(.5X+2Z)-(4X+10Z)*2}{(5X+2Z)^2} = -3X/(5X+2Z)^2$$

3.1) $Y=X + X*Z - Z^2$
$dY=dX + ZdX + XdZ-2*ZdZ$
$=(1+Z)dX + (X-2Z)dZ$

3.2) $Y=40 + 3X + 4XZ^2$
$dY=3dX + 4Z^2dX + 8XZdZ$
$= (3+4Z^2)dX +8XZdZ$

3.3) **For 3.1), solve for the numerical value of dY if X is 10 and changes to 11 and Z is 4 and changes to 4.5.**

$dY=(1+4)*1 + (10-2*4)*.5=5 + (2*.5)=6$

4.1) $Cost=8L^2-32L$
$\partial Cost/\partial L = 16L-32=0$
$L^*=2$
$\partial^2 Cost/\partial L^2=16$ **which is >0, so are at a minimum.**

4.2) Profit=-Q^3 + 57 Q^2 –315Q-2000 (solve for Q where ∂Profit/∂Q=0 and check the second order derivative for max or min).

∂Profit/∂Q = -3Q^2 + 114Q-315=0

Hint on 4.2) use quadratic formula

For $ax^2 + bx+c=0$
To find x values:

$$\frac{-b \pm (b^2 - 4ac)^{.5}}{2a}$$

$\frac{-114 +/- [114^2-4(-3)-213)]^{.5}}{-6} = \frac{-114 +/- 96}{-6}$ =Q = 3 or 35

∂^{2}Profit/∂Q^2=-6Q+114

at 3 ∂^{2}Profit/∂Q^2=-6Q+114=96 >0 so a minimum

at 35 ∂^{2}Profit/∂Q^2=-6Q+114=-96 <0 so a maximum

Q*=35

4.3) Y=50X + 25Z +5XZ

Subject to:
60=X+Z

L=50X + 25Z +5XZ + λ(60-X-Z)

First order conditions
∂L/∂X=50 + 5Z-ë=0
∂L/∂X =25+5X- ë=0
∂L/∂ë =65-X-Z=0

5+Z=X
65=5+Z+Z
60=2Z
Z=30
X=35